11/6
2026

Volume Benda Putar

① EAS 2025

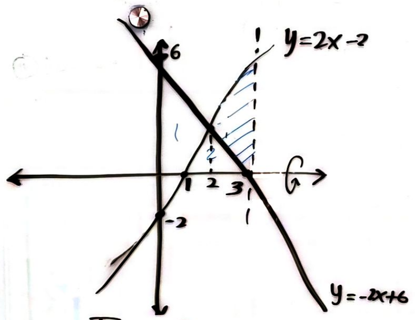
Volume benda padat dibatasi :

$y = 2x - 2$; $y = -2x + 6$; $x = 3$

diputar pada Sumbu-x.

Jawab : 1. Gambar Grafik

- $y = 2x - 2$
  - $y = 0 \rightarrow x = 1$ $(1,0)$
  - $x = 0 \rightarrow y = -2$ $(0,-2)$
- $y = -2x + 6$
  - $y = 0 \rightarrow x = 3$ $(3,0)$
  - $x = 0 \rightarrow y = 6$ $(0,6)$



2. Titik Potong

$$y_1 = y_2$$
$$2x - 2 = -2x + 6$$
$$4x = 8$$
$$x = 2$$

3. Volume

$$V = \int_2^3 \pi\left[(2x-2)^2 - (-2x+6)^2\right] dx$$

$$V = \pi \int_2^3 \left[4x^2 - 8x + 4 - (4x^2 - 24x + 36)\right] dx$$

$$= \pi \int_2^3 16x - 32 \, dx$$

$$= \pi \cdot 16 \int_2^3 x - 2 \, dx$$

$$= 16\pi \left[\tfrac{1}{2}x^2 - 2x\right]_2^3$$

$$= 16\pi \left[\tfrac{9}{2} - 6 - (2 - 4)\right]$$

$$= 16\pi \left[\tfrac{9}{2} - 4\right]$$

$$= 16\pi \left[\tfrac{1}{2}\right]$$

$$= 8\pi \text{ Satuan Volume}$$

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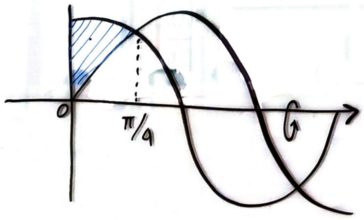
(2) Kuis 2

Vol. benda putar : $y = \cos x$; $y = \sin x$; $x = 0$; $x = \frac{\pi}{4}$

diputar thd Sb-x

Jawab :



$$V = \int_0^{\pi/4} \pi \left[\cos^2 x - \sin^2 x\right] dx$$

$$= \pi \int_0^{\pi/4} \cos 2x \, dx$$

$$= \pi \cdot \frac{1}{2} \cdot \sin 2x \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2} \cdot \left[\sin \frac{\pi}{2} - \sin 0\right]$$

$$= \frac{\pi}{2} \left[1 - 0\right]$$

$$= \frac{\pi}{2} \text{ Satuan Volume}$$

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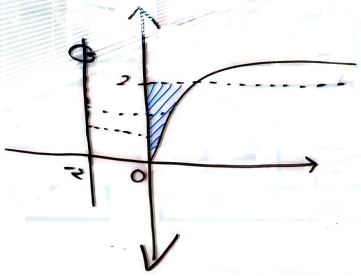
Volume Benda Putar

(3) EAS 2024

Vol. benda putar : $y=\sqrt{x}$, $y=2$, $x=0$:
diputar pada garis $x=-2$!

Jawab :

1). Sketsa Grafik $\quad x = y^2$



$$V = \int_0^2 \pi \left[[f(y)+2]^2 - 2^2 \right] dy$$

$$= \pi \int_0^2 \left[(y^2+2)^2 - 2^2 \right] dy$$

$$= \pi \int_0^2 \left[(y^2+2+2)(y^2+2-2) \right] dy$$

$$= \pi \int_0^2 \left[(y^2+4)\, y^2 \right] dy$$

$$= \pi \int_0^2 y^4 + 4y^2 \, dy$$

$$= \pi . \left[\frac{1}{5} y^5 + \frac{4}{3} y^3 \right]_0^2$$

$$= \pi \left[\frac{32}{5} + \frac{32}{3} \right]$$

$$= \pi \left[\text{banyak} \right)$$

Panjang Kurva

(4) Buku 4.3

Panjang busur kurva :

$$y = \frac{x^4}{16} + \frac{1}{2x^2}$$

$x = 2$ sampai $x = 3$

Jawab :

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

-> $f'(x)$

$$f'(x) = \frac{4}{16}x^3 + \frac{1}{2}\cdot(-2)\,x^{-3}$$
$$= \frac{1}{4}x^3 - x^{-3}$$

-> $[f'(x)]^2$

$$\left(\frac{1}{4}x^3 - x^{-3}\right)\left(\frac{1}{4}x^3 - x^{-3}\right)$$
$$= \frac{1}{16}x^6 - \frac{1}{\cancel{4}\,2}\cancel{2} + x^{-6}$$
$$= \frac{1}{16}x^6 - \frac{1}{2} + x^{-6}$$

-> Panjang Kurva

$$S = \int_2^3 \sqrt{1 + \frac{1}{16}x^6 - \frac{1}{2} + x^{-6}}\, dx$$
$$= \int_2^3 \sqrt{\frac{1}{16}x^6 + \frac{1}{2} + x^{-6}}\, dx$$

Ubah ke kuadrat sempurna

$$\left(\frac{1}{4}x^3 + x^{-3}\right)^2 =$$
$$= \frac{1}{16}x^6 + \frac{1}{2} + x^{-6}$$

$$S = \int_2^3 \sqrt{\left(\frac{1}{4}x^3 + x^{-3}\right)^2}\, dx$$
$$= \int_2^3 \frac{1}{4}x^3 + x^{-3}\, dx$$
$$= \frac{1}{4}\cdot\frac{1}{4}x^4 - \frac{1}{2}x^{-2}\Big|_2^3$$
$$= \frac{1}{16}\,81 - \frac{1}{2}\cdot\frac{1}{9} - \left(\frac{1}{16}\cdot 16 - \frac{1}{2}\cdot\frac{1}{4}\right)$$
$$= \frac{81}{16} - \frac{1}{18} - 1 + \frac{1}{8}$$

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Panjang Kurva (Parametrik)

(5) EAS 2023

Panjang busur kurva: $x = t^2$;

$y = \frac{1}{3}t^3$, interval $0 \le t \le 1$

Jawab: $S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\Rightarrow \frac{dx}{dt} = 2t$; $\frac{dy}{dt} = t^2$

$\left[\frac{dx}{dt}\right]^2 = 4t^2$; $\left[\frac{dy}{dt}\right]^2 = t^4$

$S = \int_0^1 \sqrt{4t^2 + t^4}\, dt$

$= \int_0^1 \sqrt{t^2(4+t^2)}\, dt$

$= \int_0^1 t\sqrt{4+t^2}\, dt$

※ misal: $u = 4+t^2$ | BA: $u = 5$

$du = 2t\, dt$ | BB: $u = 4$

$dt = \frac{du}{2t}$

$= \int_4^5 \cancel{t}\sqrt{u} \cdot \frac{du}{2\cancel{t}}$

$= \frac{1}{2}\int_4^5 \sqrt{u}\, du$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u\sqrt{u} \Big|_4^5$

$= \frac{1}{3}\left[5\sqrt{5} - 4\sqrt{4}\right]$

$= \frac{1}{3}\left[5\sqrt{5} - 8\right]$ Satuan panjang

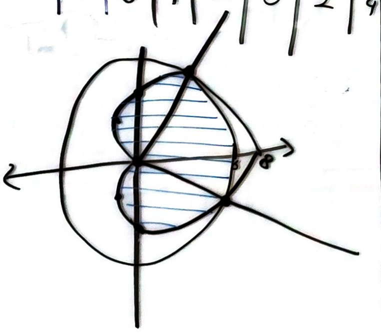
Luas Koordinat Kutub

⑥ EAS 2025

Luas daerah dalam $r=6$; dalam Kardioda $r = 4 + 4\cos\theta$

1). Sketsa Grafik

θ	0	60	90	120	180	240	270
r	8	6	4	2	0	2	4

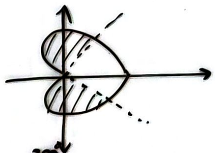


2). Titik Potong

$$r_1 = r_2$$
$$4 + 4\cos\theta = 6$$
$$4\cos\theta = 2$$
$$\cos\theta = \frac{1}{2} \rightarrow \theta = 60^\circ, 300^\circ$$

3). Tentukan Luas

①



$$L = \int_{60^\circ}^{300^\circ} \frac{1}{2}\left[(4+4\cos\theta)^2\right] d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5}{3}\pi} 16 + 32\cos\theta + 16\cos^2\theta \, d\theta$$

$$= \int_{\pi/3}^{\frac{5\pi}{3}} 8 + 16\cos\theta + 8\cos^2\theta \, d\theta$$

※ $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$= \int_{\pi/3}^{\frac{5\pi}{3}} 8 + 16\cos\theta + 8 \cdot \frac{1}{2}(1+\cos 2\theta) \, d\theta$$

$$= \int_{\pi/3}^{5\pi/3} 12 + 16\cos\theta + 4\cos 2\theta \, d\theta$$

$$= 12\theta + 16\sin\theta + 4 \cdot \frac{1}{2}\sin 2\theta \Big|_{\pi/3}^{5\pi/3}$$

$$= 12 \cdot \frac{5\pi}{3} + 16\left(-\frac{1}{2}\sqrt{3}\right) + 2\left(-\frac{1}{2}\sqrt{3}\right) - \left[12 \cdot \frac{\pi}{3} + 16 \cdot \frac{1}{2}\sqrt{3} + 2\left(\frac{1}{2}\sqrt{3}\right)\right]$$

$$= 20\pi - 4\pi - 8\sqrt{3} - \sqrt{3} - 8\sqrt{3} - \sqrt{3}$$

$$= 16\pi - 18\sqrt{3} \text{ Satuan luas}$$

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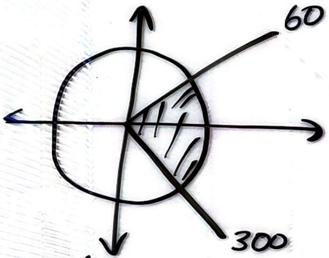
(2)



$$L = 2 \int_{0}^{60} \frac{1}{2} (6)^2 \, d\theta$$

$$= \int_{0^\circ}^{60^\circ} 36 \, d\theta$$

$$= 36 \cdot \theta \Big]_0^{\pi/3}$$

$$= 36 \cdot \frac{\pi}{3}$$

$$= 12\pi$$ Satuan luas

4). Luas Total

$$L_{tot} = L_1 + L_2$$

$$= 16\pi - 18\sqrt{3} + 12\pi$$

$$= 28\pi - 18\sqrt{3}$$ Sat. Luas

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Persamaan Garis Singgung

⑧ EAS 2024

Persamaan Parametrik: $x = t^2 + 1$

$y = t$ untuk $0 \le t \le 5$

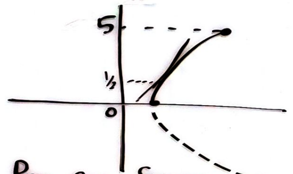
a. Sketsa kurva dengan eliminasi parameter

b. Pers. Garis Singgung di $t = \frac{1}{2}$

Jawab:

a. $x = t^2 + 1 \quad y = t$

$x = y^2 + 1, \quad 0 \le y \le 5$



b. Pers. Garis Singgung di $t = \frac{1}{2}$

→ Gradien

$m = \frac{dy}{dx} = \frac{1}{2t}$

$= \frac{1}{2(\frac{1}{2})}$

$= 1$

→ Titik Singgung di $t = \frac{1}{2}$

$x = t^2 + 1 = \left(\frac{1}{2}\right)^2 + 1$

$= \frac{5}{4}$

$y = t \quad = \frac{1}{2}$

$(x, y) = \left(\frac{5}{4}, \frac{1}{2}\right)$

→ PGS

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = 1\left(x - \frac{5}{4}\right)$

$y = x - \frac{5}{4} + \frac{1}{2}$

$y = x - \frac{3}{4}$

$4y - 4x + 3 = 0$

Garis Singgung – Kutub

g). Dapatkan semua Koordinat di limacon $r = 1 - \sin\theta$ dengan garis singgung Vertikal

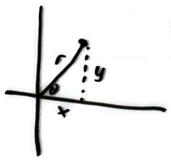
Jawab :

→ Garis Singgung Vertikal :

- Pembilang $\neq 0$ ✓
- Penyebut $= 0$

$$m = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

→ Koordinat Kutub



$x = r\cos\theta$

$y = r\sin\theta$

→ $x = (1 - \sin\theta) \cdot \cos\theta$

$$\frac{dx}{d\theta} = -\cos\theta \cdot \cos\theta + (1 - \sin\theta)(-\sin\theta)$$

$$= -\cos^2\theta - \sin\theta + \sin^2\theta$$

→ $y = (1 - \sin\theta) \cdot \sin\theta$

$$\frac{dy}{d\theta} = -\cos\theta \cdot \sin\theta + (1 - \sin\theta)\cos\theta$$

$$= -\cos\theta\sin\theta + \cos\theta - \sin\theta\cos\theta$$

$$= -2\sin\theta\cos\theta + \cos\theta$$

• Cari penyebut $= 0$

$$-\cos^2\theta + \sin^2\theta - \sin\theta = 0$$

$$(\sin^2\theta - 1) + \sin^2\theta - \sin\theta = 0$$

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$\sin\theta = -\tfrac{1}{2} \qquad \sin\theta = 1$$

$\theta = 210°, 330°, 90°$

• Cek Pembilang

$\theta = 210° \rightarrow \frac{dy}{d\theta} = \dots$ ✓ $\quad \theta = 330°$

$\theta = 90° \rightarrow \frac{dy}{d\theta} = 0$ ✗

→ Titik Koordinat

• $210° \rightarrow r = 1 - \sin 210 = \frac{3}{2}$ $\quad \left(\frac{3}{2}, 210°\right)$

• $\theta = 330° \rightarrow r = 1 - \sin 330° = \frac{3}{2}$ $\quad \left(\frac{3}{2}, 330°\right)$